IV Semester M.Sc. Degree Examination, June/July 2014
(RNS) (2012-13 and Onwards)
MATHEMATICS
M-402 : Numerical Analysis and Matlab/Scilab Programming – II

Time : 3 Hours Max. Marks : 60

Instructions : *Answer **any five full** questions choosing atleast **one** from each Part.*

PART – A

1. a) Derive the classical Runge-Kutta explicit method of two slopes for solving.

$$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0.$$ 6

b) Suggest a suitable range of step size to solve

$$\frac{dy}{dx} = -4y;\ y(0) = 1.$$

by classical Runge-Kutta explicit method of two slopes. 6

2. a) Derive any one Adam-Bashforth-Moulton predictor-corrector method to solve

$$\frac{dy}{dx} = f(x, y);\ y(x_0) = y_0.$$ 6

b) Solve $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0;\ y(\pm 1) = 0$ by a second-order finite difference approximation. Do 4 iterations of the Gauss-Siedel method for solving the resulting system of linear algebraic equations. Choose $\Delta x = 0.25$. 6

PART – B

3. a) Solve the following Dirichlet problem by finite difference method.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0;\ 0 \le x \le 1,\ 0 \le y \le 1.$$

Subject to

$$\left.\begin{array}{l} u(x, 0) = x \\ u(x, 1) = 0 \end{array}\right\},\ 0 \le x \le 1$$

$$\left.\begin{array}{l} u(0, y) = 0 \\ u(1, y) = 0 \end{array}\right\},\ 0 \le y \le 1$$

Choose $\Delta x = \Delta y = \frac{1}{3}$. 6

P.T.O.

b) Using the central difference approximation solve the BVP.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2 \; ; \; 0 \le x, y \le 3.$$

Subject to

$u(x, 0) = 3x$, $\quad 0 \le x \le 3$

$u(x, 3) = 0$, $\quad 0 \le x \le 2$

$u(0, y) = 0$, $\quad 0 \le y \le 3$

$u(x, y) = 9 - 3y$ on $y = -3x + 9$, $2 \le x \le 3$. **6**

4. a) Using Schmidt explicit finite difference method solve the IBVP :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \; ; \; 0 \le x \le 1, t \ge 0.$$

Subject to

$u(x, 0) = \sin(\pi x)$, $0 \le x \le 1$

$$\left.\begin{array}{l} u(0, t) = 0 \\ u(1, t) = 0 \end{array}\right\} ; \; 0 \le t < \infty.$$

Choose $\Delta x = \frac{1}{4}$ and using the stability criterion choose an appropriate value of Δt. Obtain the solution at the second time level. **6**

b) Prove or disprove the statement that Crank-Nicolson finite difference method of solving one-dimensional heat equation is unconditionally stable. **6**

5. a) With the help of an explicit finite difference approximation obtain an approximate solution of the IBVP.

$$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2} \; ; \; 0 \le x \le 1, t \ge 0.$$

Subject to

$$\left.\begin{array}{l} u(x, 0) = \sin(\pi x) \\ \frac{\partial u}{\partial t}(x, 0) = 0 \end{array}\right\} ; \; 0 \le x \le 1.$$

$$\left.\begin{array}{l} u(0, t) = 0 \\ u(1, t) = 0 \end{array}\right\} , \; t \ge 0.$$

Choose $\Delta x = \frac{1}{4}$ and an acceptable value of Δt. Obtain the solution at the second time level. **6**

b) Show that the explicit finite difference method of solving the one-dimensional wave equation is conditionally stable. **6**

PART – C

6. a) Explain through simple examples mathematical operations with arrays in matlab or scilab. **4**

   b) Illustrate the usage of two-dimensional and three dimensional plots in matlab or scilab with suitable examples. **4**

   c) Write a matlab or scilab program to create a 4 × 4 matrix A of real elements and using built-in functions find the following :

      i) Determinant of A, ii) Transpose of A,

      iii) Rank of A and iv) Eigenvalues of A **4**

7. Write a matlab or scilab program to solve

   a) Laplace equation and

   b) One-dimensional heat equation by central difference approximation. **(6+6)**

8. a) Implement the classical explicit Runge-Kutta method of two-slopes for solving $\frac{dy}{dx} = x + y^2$; y(0) = 1, in matlab or scilab. Assume $\Delta x = 0.05$ and the value of y to be obtained at x = 0.1. **6**

   b) Write a matlab or scilab program to implement shooting method, based on any Runge-Kutta explicit method, for solving a linear BVP, (ODE) of 2nd order. **6**